Computationally Efficient TDOA/FDOA Based Moving Target Localization for a Distributed Sensor Network

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ABSTRACT

We consider the problem of estimating the location and velocity of a moving target using a distributed sensor network. We first present the maximum likelihood estimator (MLE) using received signals, when the source signal is unknown and modelled as a deterministic process. Since the MLE requires a multi-dimensional search and is computationally intensive, we also develop an efficient algorithm using a two-step approach. The first step finds the time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA) estimates of each sensor with respect to a reference sensor by using a 2-dimensional fast Fourier transform, while the second step employs an iterative re-weighted least squares (IRLS) approach with a varying weighting matrix to determine the target location and velocity. Numerical results show that the IRLS approach has a lower signal-to-noise ratio (SNR) threshold compared with a recent TDOA/FDOA-based method, especially when the target is considerably farther away from some sensors than others, which creates a larger disparity in the quality of radar observations.

1.0 INTRODUCTION

Target localization is a fundamental signal processing problem encountered in a wide range of sensing and surveillance applications. Maximum likelihood estimation based on a suitable coherent signal model is a popular approach for developing high-resolution localization solutions. One such method, based on a coherent delay and Doppler model, was introduced in [1], which estimates the location of the target directly from the signal measurements by assuming the target signal is a stochastic process with known statistics. When the target is moving, time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA) measurements can be utilized to determine the target location and velocity. However, since both the TDOA and FDOA are non-linearly dependent on the target location and velocity, localizing a moving target from TDOA and FODA measurements is a challenging problem. One way to deal with the non-linearity is to introduce redundant parameters in the TDOA and FDOA versus target location and velocity relation. An algebraic solution for the position and velocity of the moving target was proposed in [2] by employing redundant parameters to linearize the non-linear estimation problem. However, the redundant parameter-based approach leads to considerable bias caused by the noise correlation between the regressor and regressand in the formulation. To address the issue, an extension of [2] was presented in [3], which exploits a relation between the redundant parameters and the target position/velocity to refine the solution. Another approach is to directly solve the non-linear estimation problem by using non-linear optimization methods [4], [5]. For example, [5] solved the non-linear localization problem in two steps. In each step, a non-linear weighted least squares problem using TDOA estimates (first step), or both TDOA and FDOA estimates (second step), was formulated and solved, followed by bias reduction.

Most TDOA/FDOA based methods require the covariance matrix of the TDOA and FDOA estimates for weighted least squares fitting. The covariance matrix is often unknown in practice since it depends on radar-to-target distances and signal/noise statistics. In this paper, we consider moving target localization using passive radar. We first consider a direct signal-based approach and derive the maximum likelihood estimator (MLE) of the target location and velocity, assuming the target waveform is unknown and modeled as a deterministic process. The MLE obtains the target location and velocity estimates through a search procedure over the parameter space. While asymptotically optimum, the MLE may be practically infeasible in some scenarios with a large number of observations due to its complexity. To address the issue, we propose a computationally more efficient two-step method based on TDOA/FDOA estimates. In the first step, we obtain the TDOA and FDOA estimates from the signal measurements by using a two-dimensional (2-D) fast Fourier transform (FFT). In the second step, we use an iterative reweighted least squares (IRLS) process to find the location and velocity of the target from the TDOA and FDOA estimates, each iteration involving a closed-form update of the parameters. The IRLS is seen to usually converge in a few iterations.

2.0 DATA MODEL

We consider a distributed sensor radar network, where M widely separated sensors are utilized to receive the signal reflected by a target which is located on a 2-D plane at $\mathbf{u}=[x,y]^T$ and moving with a velocity $\mathbf{v}=[v_x,v_y]^T$. The coordinates of the m-th sensor are (x_m,y_m), $m=1,2,\ldots,M$. Assume the first sensor is selected as the reference sensor. Let $\mathbf{s}\in\mathbb{C}^{N\times 1}$ denote the unknown source waveform observed at senor 1, where N denotes the number of samples obtained over the observation window. Then the signals observed at the other sensors over the same observation window can be written as:

$$\mathbf{r}_m=\alpha_m\mathbf{\Phi}(\tau_{m1},f_{m1})\mathbf{s}+\mathbf{w}_m,\qquad m=2,\ldots,M, \tag{1}$$

where α_m denotes the target amplitude that integrates the radar cross section (RCS), the antenna gain and channel propagation attenuation, $\mathbf{\Phi}$ is an $N\times N$ delay and Doppler shifting matrix controlled by the TDOA $\tau_{m1}=\tau_m-\tau_1$ and FDOA $f_{m1}=f_m-f_1$, and $\mathbf{w}_m$ denotes the observation noise which is assumed zero-mean and Gaussian with covariance matrix $\sigma^2\mathbf{I}$. Here, τ_m and f_m denote the time of arrival and Doppler frequency observed by the m-th sensor. The delay/Doppler shifting matrix can be expressed as [6]: $\mathbf{\Phi}(\tau_{m1},f_{m1})=\mathbf{W}(f_{m1}T_s)\mathbf{T}^H\mathbf{W}(-\tau_{m1}\Delta f)\mathbf{T}$, where $\mathbf{T}$ denotes the $N\times N$ unitary discrete Fourier transform (DFT) matrix while $\mathbf{W}(a)$ is a diagonal matrix with the n-th diagonal entry given by $e^{j2\pi(n-1)a}$. The problem of interest is to estimate the location $\mathbf{u}$ and the velocity $\mathbf{v}$ of the moving target by using observations from the M sensors.

3.0 PROPOSED APPROACHES

We first present the maximum likelihood estimator (MLE), which is optimum but computationally costly, and then propose an efficient iteratively re-weighed least squares (IRLS) method based on TDOAs and FDOAs.

3.1 MLE

The MLE can be obtained by jointly maximizing the likelihood function of the received signals $[\mathbf{r}_1^T,\mathbf{r}_2^T,\ldots,\mathbf{r}_M^T]$ with respect to the unknown parameters, which include the target location/velocity $\mathbf{u}$ and

$\mathbf{v}$, target amplitude $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_M]$, source waveform $\mathbf{s}$, and noise variance σ^2. It can be shown the MLE of $\mathbf{u}$ and $\mathbf{v}$ can be found by maximizing the largest eigvenvalue of the Gram matrix $\mathbf{Z}^H\mathbf{Z}$:

$$\{\hat{\mathbf{u}}, \hat{\mathbf{v}}\} = \arg\max_{\mathbf{u},\mathbf{v}} \lambda_{\max}(\mathbf{Z}^H\mathbf{Z}), \tag{2}$$

where $\mathbf{Z} = [\mathbf{r}_1 \;\; \boldsymbol{\Phi}_{21}^H\mathbf{r}_2 \;\; \cdots \;\; \boldsymbol{\Phi}_{M1}^H\mathbf{r}_M]$, The maximization requires a 4-dimensional (4-D) search, which is computationally intensive. Once the estimates of $\mathbf{u}$ and $\mathbf{v}$ are found, the other parameters can be estimated fairly easily. Specifically, $\mathbf{s}$ is given by the eigenvector of $\mathbf{Z}^H\mathbf{Z}$ associated with the largest eigenvector, while estimates of $\boldsymbol{\alpha}$ and σ^2 can be obtained by least squares.

3.2 Iteratively Re-weighed Least Squares (IRLS)

The proposed IRLS algorithm uses the TDOA and FDOA measurements of sensors 2 to M with respect to the reference sensor 1. The TDOA and FDOA can be efficiently computed by using the 2-D FFT based approach introduced in [6]. Let the resulting TDOA and FDOA estimated be denoted by $\hat{\tau}_{m1}$ and $\hat{f}_{m1}$. Consider the *range difference* d_{m1} between the m-th sensor and the reference sensor, which can be determined from the TDOA:

$$d_{m1} = \sqrt{(x - x_m)^2 + (y - y_m)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} = c(\tau_m - \tau_1) = c\tau_{m1}, \quad m = 2,3,\ldots,M. \tag{3}$$

Let $\hat{\mathbf{d}} = [\hat{d}_{21}, \hat{d}_{31}, \ldots, \hat{d}_{M1}]^T$ which contain measured range differences obtained from the TDOA estimates. We have

$$\hat{\mathbf{d}} = \mathbf{g}(\mathbf{u}) + \mathbf{e}_u, \tag{4}$$

where $\mathbf{e}_\mathbf{u}$ is the range difference estimation error for the range difference and $\mathbf{g}(\mathbf{u}) = [g_2(\mathbf{u}), g_3(\mathbf{u}), \ldots, g_M(\mathbf{u})]^T$ is the noise-free range difference $g_m(\mathbf{u}) = d_{m1}$, which is a non-linear function of the target location parameter $\mathbf{u}$. Let $\mathbf{u}^{(l-1)} = [x^{(l-1)}, y^{(l-1)}]^T$ denote the location estimate obtained from the $(l-1)$-th iteration. Applying the first-order Taylor expansion of $\mathbf{g}(\mathbf{u})$ at $\mathbf{u}^{(l-1)}$ yields

$$\hat{\mathbf{d}} \approx \mathbf{g}(\mathbf{u}^{(l-1)}) + \mathbf{G}_p^{(l-1)}\Delta\mathbf{u}^{(l)} + \mathbf{e}_u, \tag{5}$$

where $\Delta\mathbf{u}^{(l)} = [\Delta x^{(l)} \;\; \Delta y^{(l)}]^T$ and $\mathbf{G}_p^{(l-1)}$ denotes the $(M-1)\times 2$ Jacobian matrix of $\mathbf{g}(\mathbf{u})$ computed at $\mathbf{u}^{(l-1)}$. It follows from the linear model that $\Delta\mathbf{u}^{(l)}$ can be obtained by a weighted least squares fitting

$$\Delta\mathbf{u}^{(l)} = \left(\mathbf{G}_p^{(l-1)H}\mathbf{R}_u^{(l-1)}\mathbf{G}_p^{(l-1)}\right)^{-1}\mathbf{G}_p^{(l-1)H}\mathbf{R}_u^{(l-1)}\left(\hat{\mathbf{d}} - \mathbf{g}(\mathbf{u}^{(l-1)})\right), \tag{6}$$

where $\mathbf{R}_u^{(l-1)}$ is a weighting matrix. We employ a varying diagonal weighting matrix with diagonal elements given by $[\hat{d}_{m1} - \mathbf{g}_m(\mathbf{u}^{(l-1)})]^{-2}$. Once $\Delta\mathbf{u}^{(l)}$ is obtained, the target location is updated by $\mathbf{u}^{(l)} = \mathbf{u}^{(l-1)} + \Delta\mathbf{u}^{(l)}$. The iterative process ends when $\Delta\mathbf{u}^{(l)}$ is smaller than a pre-specified tolerance level ε.

Once the target location estimate is obtained, we can find the target velocity estimate by utilizing a similar iterative reweighted procedure. Let $\dot{d}_{m1}$ denote the *range rate difference* between the m-th sebsir and the reference, which can be determined from the FDOA:

$$\dot{d}_{m1} = \frac{\partial d_m}{\partial t} - \frac{\partial d_1}{\partial t} = \frac{v_x(x-x_m)+v_y(y-y_m)}{d_m} - \frac{v_x(x-x_1)+v_y(y-y_1)}{d_1} = -\lambda f_{m1}, \qquad m = 2,,\ldots,M. \tag{7}$$

Let $\hat{\dot{\mathbf{d}}} = [\hat{\dot{d}}_{21}, \hat{\dot{d}}_{31}, \ldots, \hat{\dot{d}}_{M1}]^T$ contain range rate difference vector obtained from FDOA estimates. We have

$$\hat{\dot{\mathbf{d}}} = \mathbf{H_u v} + \mathbf{e}_v, \tag{8}$$

where $\mathbf{e}_v$ is the estimation error for the range rate difference vector $\hat{\dot{\mathbf{d}}}$ and $\mathbf{H_u}$ is an $(M-1)\times 2$ matrix with the $(m-1)$-th given by $\left[\frac{x-x_m}{d_m} - \frac{x-x_1}{d_1} \quad \frac{y-y_m}{d_m} - \frac{y-y_1}{d_1}\right]$, $m = 2,\ldots,M$. We use the target location estimate $\hat{\mathbf{u}}$ to form $\hat{\mathbf{H}}_\mathbf{u}$. Let $\mathbf{v}^{(l-1)} = [v_x^{(l-1)}, v_y^{(l-1)}]^T$ denote the results from the $(l-1)$-th iteration. We obtain

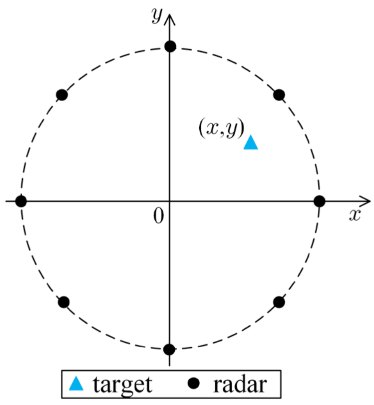
$$\mathbf{v}^{(l)} = [\hat{\mathbf{H}}_\mathbf{u}^H \mathbf{R}_\mathbf{v}^{(\mathbf{l-1})} \hat{\mathbf{H}}_\mathbf{u}]^{-1} \hat{\mathbf{H}}_\mathbf{u} \mathbf{R}_\mathbf{v}^{(\mathbf{l-1})} \hat{\dot{\mathbf{d}}}, \tag{9}$$

where $\mathbf{R}_\mathbf{v}^{(\mathbf{l-1})}$ is the diagonal weighting matrix with diagonal elements $[(\hat{\dot{d}}_{m1} - \hat{\mathbf{H}}_\mathbf{u}(m,:)\mathbf{v}^{(l-1)}]^{-2}$, with $\hat{\mathbf{H}}_\mathbf{u}(\mathrm{m},:)$ denoting the m-th row of matrix $\hat{\mathbf{H}}_\mathbf{u}$. The iteration of finding the estimate of the target velocity $\hat{\mathbf{v}}$ ends when $\mathbf{v}^{(l)} - \mathbf{v}^{(l-1)}$ is smaller than the tolerance level ε.

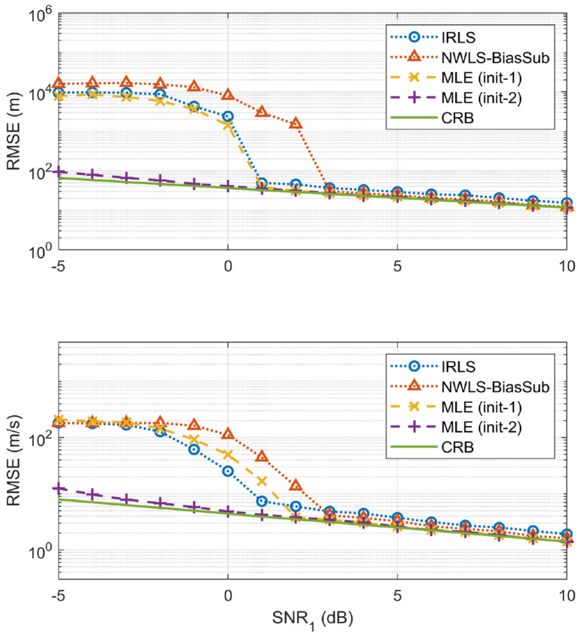
Remark: Due to different sensor-to-target distances, the measurements of the TDOAs and FDOAs are of different quality. It is crucial to take such differences into account in estimating the target location and velocity. A standard approach to dealing with this problem is to use the inverse of the covariance matrix of the TDOA and FDOA estimates as a weighting matrix (e.g., [2]). In practice, the covariance matrix depends on the sensor-to-target geometry and is usually unknown a priori. To address this issue, we employ practical weighting matrices (6) and (9) that are readily computable. Intuitively, if the m-th sensor is close to the target, it is expected that the range difference fitting error $\hat{d}_{m1} - g_m(\mathbf{u})$ is small, and so is the range rate difference fitting error. As such a larger weight is applied to this sensor than those with poorer measurement quality. Through this process, the proposed iterative reweighted scheme is able to perform automatic sensor selection.

4.0 SIMULATION RESULTS

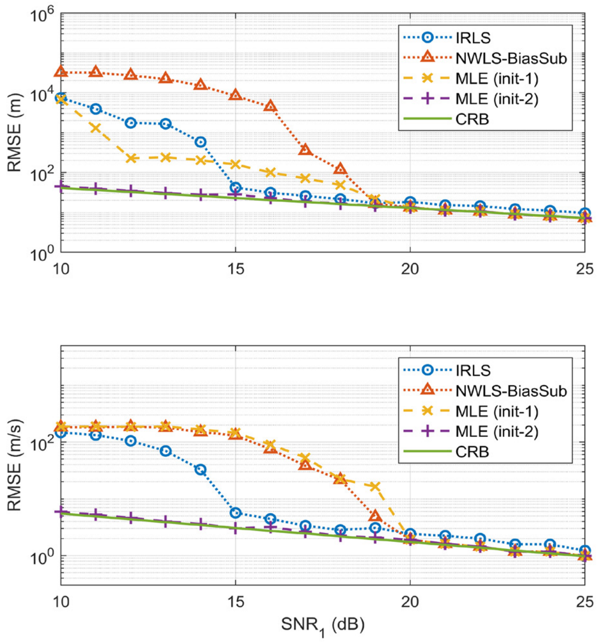
We consider a distributed radar network depicted in Figure 1(a) that consists of $M = 8$ sensors uniformly spaced on a circle with a radius of $33.33cT_s$, where c and T_s denote the signal propagation speed and sampling interval, respectively.



(a)



(b)



(c)

Figure 1: (a) A distributed radar network with M = 8 sensors. Location (upper) and velocity (lower) estimate versus SNR$_r$ for (b) Case 1, and (c) Case 2.

The signal-to-noise ratio (SNR) for the m-th sensor is defined as: $\mathrm{SNR}_m = \frac{|\alpha_m|^2}{\sigma^2} = \frac{d_1^2}{d_m^2}\mathrm{SNR}_r$, where d_m is the distance from the target to the m-th sensor and SNR_r is the SNR for the reference sensor (sensor 1). Two cases are considered. In Case 1 (*small SNR spread*), the target is located at $\mathbf{u} = [4cT_s, 6cT_s]^T$, which is relatively close to the center of the circle. This results in similar distances between different sensors and the target and, in turn, a small SNR spread for all sensors. In Case 2 (*large SNR spread*), the target is located at $\mathbf{u} = [19.33cT_s, 23.33cT_s]^T$, which is close to one particular sensor (the reference), leading to a larger SNR spread among all sensors. Moreover, for both cases, we assume the target is moving with a velocity $\mathbf{v} = [6\times10^{-7}c, 6.66\times10^{-7}c]^T$. In the simulation, we assume $c = 3\times10^8\, m/s$, carrier frequency is 3 GHz, and the sampling frequency $1/T_s = 200$ kHz.

We compare the IRLS with the MLE in and NWLS-BiasSub method [5]. Two versions of the MLE with different initialization schemes are considered. For MLE (init-1), the initial target location/velocity is obtained by intersecting the hyperbolas associated with two best TDOA measurements along with least-squares estimate of the velocity based on (8), while MLE (init-2) initialized by the true target location and velocity. In addition, the Cramer-Rao Bound (CRB) is included in the comparison, to benchmark the estimation performance.

Figure 1(b) shows the root-mean-square error (RMSE) of the above methods along with the CRB versus SNR for the reference sensor in Case 1. It is seen that for both location and velocity estimation, there is a threshold effect, whereby the RMSE is far away from the CRB until the SNR is above a threshold [12]. The results show that all methods approach the CRB at the high SNR region, but with different thresholds. For example, the proposed IRLS is about 2 dB better than NWLS-BiasSub for both location and velocity estimation. MLE (init-1) is similar to IRLS for location estimation, but has a higher threshold than IRLS for velocity estimation. This is because MLE is a search-based method subject to local convergence caused by inaccurate initialization. With ideal (but impractical) initialization, MLE (init-2) is the best among all methods. Figure 1(c) shows the results in Case 2. With a larger SNR spread, i.e., when the sensors are more different in terms of measurement quality, the proposed IRLS enjoys a larger benefit in SNR threshold, which is reduced by 4 to 5 dB compared with NWLS-BiasSub. MLE (init-1) appears to experience a more severe local convergence problem, yielding a significantly higher threshold than that of IRLS for both location and velocity estimation.

5.0 CONCLUSION

We examined moving target localization using a distributed sensor network, where the target reflects an unknown source signal to radars. We first presented an MLE approach, based on direct received signals, which is asymptotically optimum but whose complexity grows rapidly with the observation size. For practical implementations, we also proposed a computationally more efficient TDOA/FDOA-based IRLS method that employs a 2-D FFT and iterative re-weighting method to solve the problem. Numerical results show that IRLS approach has a lower SNR threshold and compares favorably with a recent TDOA/FDOA-based solution, in particular when the SNR observed at different radars exhibits a large spread.

6.0 REFERENCES

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